

Financial Mathematics

FORMULAE FROM PREVIOUS GRADES

Simple Interest	$A = P(1 + in)$	A is the final (accumulated) amount. P is the original (principal) amount. <i>i</i> is the interest rate per period. <i>n</i> is the number of periods.
Compound Interest or Inflation	$A = P(1 + i)^n$	A is the final (accumulated) amount. P is the original (principal) amount. <i>i</i> is the interest rate per compounding period. <i>n</i> is the number of periods.
Straight Line Depreciation	$A = P(1 - in)$	A is the final (depreciated) value. P is the original value. <i>i</i> is the depreciation rate per period. <i>n</i> is the number of periods.
Reducing Balance Depreciation	$A = P(1 - i)^n$	A is the final (depreciated) value. P is the original value. <i>i</i> is the depreciation rate per period. <i>n</i> is the number of periods.
Effective and Nominal Interest Conversion	$1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m$	i_{eff} is the effective annual interest rate. i_{nom} is the nominal interest rate (rate p.a. compounded more than once a year). <i>m</i> is the number of times interest is compounded in one year.

REMINDERS:

1. Inflation:

The rate at which prices increase over time

2. Consumer Price Index (CPI):

the average prices of a basket of goods

3. Exchange Rates:

The value of one currency for the purpose of conversion to another

4. Population Growth:

Change of population size over time

5. Hire Purchase:

Short term loan, deposit payable. Calculated using simple interest.

6. Reducing balance loan:

Interest is paid on the reducing balance, the lower the balance, the less you have to pay.

7. Nominal interest rate:

Quoted period and compounded period is different eg 15% per annum compounded monthly.

8. Effective interest rate:

Quoted period and compound period is equal eg 0,75% per month compounded monthly.

COMPOUND PERIODS

Annually: 1 per year

Semi-annually: 2 per year

Quarterly: 4 per year

Monthly: 12 per year

Daily: 365 per year*
*(excl leap years)

CHANGING INTEREST RATES

If the interest rate changes after a set period of time:

1. Determine the accumulated amount after the first period
2. Set the accumulated amount as the initial amount for the second period
3. Determine the accumulated amount after the second period.

EXAMPLE

R100 000 is invested for 6 years at an interest rate of 16% per annum compounded quarterly. Thereafter the accumulated amount is reinvested for 5 years at an interest rate of 14% compounded semi-annually. Calculate the value of the investment at the end of this period.

HIRE PURCHASE

You buy a washing machine of R4 000 by signing a 2 year hire purchase agreement, you pay an R800 deposit. Calculate the

- a) total amount you will repay if the interest rate is 12%
- b) your monthly installment.

DEPRECIATION (DECAY)

Depreciation is the loss or decrease of value at a specified rate over time.

Depreciation: Loss of value over time

Book value: Value of equipment at a given time after depreciation

Scrap value: Book value of equipment at the end of its useful life

A = Book or scrap value

P = Present value

i = Depreciation rate

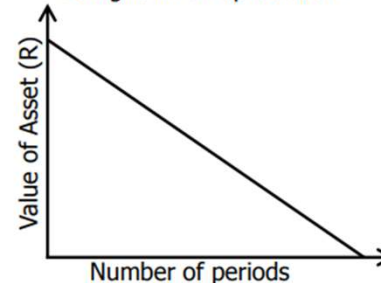
n = time period

LINEAR DEPRECIATION

Also known as simple decay or straight line depreciation

$$A = P(1 - in)$$

Straight Line Depreciation

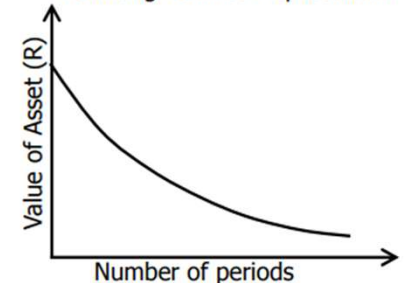


COMPOUND DEPRECIATION

Also known as depreciation on a reducing balance

$$A = P(1 - i)^n$$

Reducing Balance Depreciation



EXAMPLE

My new car, to the value of R 200 000 depreciates at a rate of 9% per annum. What would the value of my car be after 6 years? Compare a linear depreciation to a reducing balance depreciation.

FUTURE VALUE ANNUITIES

Equal investment payments made at regular intervals, subject to a rate of interest over a period of time.
Eg annuity fund, a retirement fund, a savings account.

$$F = \frac{x[(1+i)^n - 1]}{i}$$

F = total accumulated at the end of the time period
x = monthly installment
i = interest rate per annum
n = number of installments/payments

EXAMPLE

R2000 is deposited into a bank account. One month later a further R2 000 is deposited, and another R2 000 a month after that. The interest rate is 7% per annum compounded monthly. How much will be saved after 2 months?

PRESENT VALUE ANNUITIES

Reducing balance loan, a sum of money borrowed and paid back with interest in regular payments at equal intervals.

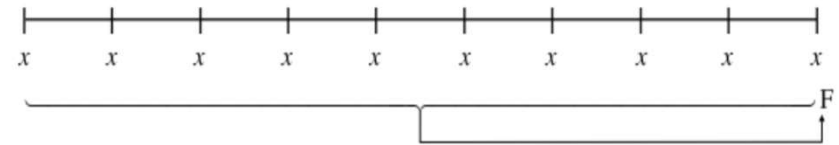
$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

P = present value
x = monthly installment
i = interest rate p.a.
n = number of time periods to repay the loan

EXAMPLE

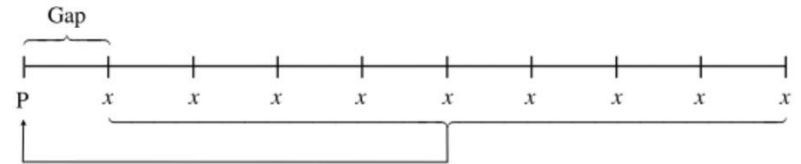
You borrow money for a year at an interest rate of 18% per annum compounded quarterly and monthly repayments will be R 1 367,20. The payments will start one month after receiving the loan. What was the initial amount borrowed?

On a timeline:



Immediately after the last payment

On a timeline:



Exactly one interval before first payment

CALCULATE MONTHLY INSTALLMENTS

Monthly installments are calculated using the present value annuity formula, and solving for x , the repayment amount.

NB: If repayments commence one month after the initiation of the loan;

- Calculate the growth of the loan during the first month to determine the new present value
- Subtract one month from the total repayment terms.

CALCULATE OUTSTANDING BALANCE

Outstanding balance is calculated using the present value annuity formula. The present value after the n^{th} installments is the outstanding balance, also known as the settlement amount.

EXAMPLE

In order to buy a car John takes a loan of R 25 000. The bank charges an annual interest rate of 11% compounded monthly. The installments start a month after he has received the money from the bank.

Calculate

- his monthly installments if he has to pay back the loan over a period of 5 years.
- the outstanding balance of his loan after two years immediately after the 24th installment.

ANALYSES OF INVESTMENT AND LOAN OPTIONS

When analysing investment and loan options, consider both the payment amounts, as well as the total payment made at completion.

Investments with higher future values are preferable.

Loans with smaller present values are preferable.

EXAMPLE

You have to take a home loan of R8 000 000 and there are 2 options to consider.

A- a 20 year loan at 17% interest per annum compounded monthly.

B- a 30 year loan at 17% interest compounded monthly.

Monthly repayments:

$$\begin{aligned} \text{A. } P &= \frac{x[1 - (1 + i)^{-n}]}{i} \\ 8\,000\,000 &= \frac{x\left[1 - \left(1 + \frac{0,17}{12}\right)^{-240}\right]}{\frac{0,17}{12}} \\ 8\,000\,000 \times \frac{0,17}{12} &= x\left[1 - \left(1 + \frac{0,17}{12}\right)^{-240}\right] \\ 11\,333,33 &= 0,9658x \\ x &= R11\,734,40 \end{aligned}$$

$$\begin{aligned} \text{B. } P &= \frac{x[1 - (1 + i)^{-n}]}{i} \\ 8\,000\,000 &= \frac{x\left[1 - \left(1 + \frac{0,17}{12}\right)^{-360}\right]}{\frac{0,17}{12}} \\ 8\,000\,000 \times \frac{0,17}{12} &= x\left[1 - \left(1 + \frac{0,17}{12}\right)^{-360}\right] \\ 11\,333,33 &= 0,99368x \\ x &= R11\,405,40 \end{aligned}$$

Total repayments:

$$A : R11\,734,40 \times 240 = R2\,816\,256$$

$$B : R11\,405,40 \times 360 = R4\,105\,944$$

Option B has a lower monthly repayment, but a total amount of almost double that of OPTION A, therefore A is the better option.